Research on Optimal Portfolio Decision Model Based on MRAT Hybrid Model

Yifan Zhao*, Nan Jiang, Ke Xu

College of Sciences, Nanjing Agricultural University, Nanjing, Jiangsu, 210095

*Corresponding author: zhaoyifan0312@163.com

Keywords: MRAT Hybrid Model, Gold, Bitcoin, ARIMA, Sub-Models.

Abstract: Market traders often aim to maximize total returns when they buy and sell assets with volatile prices. Gold has a long history as a trading tool and asset currency, while Bitcoin is a digital currency that has emerged in recent years. This paper proposes a novel quantitative trading decision-making model – MRAT Hybrid Model, to help traders make decisions in the gold and bitcoin markets. The model consists of three interlocking sub-models. They are the Bull and Bear Market Judgment Model (M), Investment Risk Model (R), and ARIMA Time Series Forecast Model (A). This paper finally proves the optimality of the model. Therefore, it can be concluded that the MRAT mixed model in this paper is the optimal portfolio decision model. The innovation of this paper mainly lies in the selection of indicators and the establishment of a mixed transaction investment model. The article splits the overall investment strategy model into three sub-models and couples them to form the final MRAT Hybrid Model.

1. Introduction

Money is a product of the times. For a long time, gold has been a tradable tool and asset currency, and the prosperity of credit currency stems from the vigorous development of technology and the economy. As a representative of digital currency, Bitcoin is called the new gold of the digital age because it is easy for traders to buy and distribute Bitcoin [1]. Bitcoin is characterized by high yields and volatility but is illiquid compared to gold. Combining gold and Bitcoin can balance risks, so it is of great research value to analyze the cointegration relationship. This article aims to maximize the total return by studying the combination of gold and bitcoin to achieve the effect of hedging.

With the rapid development and expansion of the Bitcoin market, the research literature on gold and Bitcoin has also grown. The existing literature mainly focuses on the correlation of Bitcoin with global economic activities, risk hedging ability, and impact on traditional markets to demonstrate Bitcoin's "aggression" against traditional currencies such as gold [2]. David Lubo et al. [3] address the timely question of whether Bitcoin exhibits the safe-haven properties of stock market investments under extreme market conditions and whether such properties are similar or different to gold and general commodity indices. Kyriazis, N.A. [4] found that Bitcoin can be an effective hedge against oil and stock market indices, but to a lesser extent than gold. Bitcoin and gold exhibit a low or harmful, or asymmetric nonlinear relationship. Bauer, Dirk et al. [5] employed an econometric model to replicate the findings and demonstrated that exact replication is impossible and that alternative statistical methods provide more reliable, based on the original sample and extended sample period that Bitcoin performs Significantly different returns, volatility, and correlation characteristics compared to other assets, including gold and the U.S. dollar. Unlike H Irene and S Perry [6], they used several different multivariate GARCH models, Generalized Orthogonal GARCH, to study a portfolio in Bitcoin. This paper designs some original models and utilizes the ARIMA time series forecasting model to determine the best value between USD, Gold, and Bitcoin portfolios.

2. Model analysis

The main research of this paper is to design a quantitative optimal portfolio decision model and optimize daily trading strategies based on price data. Then according to the designed model and

algorithm, the optimality of the model is verified, and the sensitivity analysis of the strategy is carried out to determine its sensitivity to different transaction costs. Finally, the impact of transaction costs on strategies and outcomes is studied. This paper firstly establishes a quantitative optimal portfolio decision model - MRAT Hybrid Model. It includes three sub-models - bull and bear market judgment, investment risk, and ARIMA series prediction.

Considering the background information and restricted conditions identified in the problem statement, we are required to design a quantitative optimal portfolio decision model to optimize daily trading strategy based on price data and offer evidence to verify the model's optimality. In addition, perform sensitivity analysis of this strategy and determine its sensitivity to different transaction costs.

3. Establishment of models

The paper decided to adopt a dynamic programming model based on circular trading rules to quantify trading decisions better. When building this model, the paper pre-set three sub-models: bull and bear markets judgment model (M), investment risk model (R), and ARIMA series prediction model (A).

3.1. MRAT Hybrid Model

On non-trading days, the paper assumes that the gold price remains unchanged and supplements the price of gold with the value of the previous trading day. Then it gets the three indicators of date D(Day), gold price P_1(Price1), and Bitcoin price P(Price2) under the same dimension.

Gold rose. It indicates how much the price of gold has changed on the current day compared to the price on the previous day.

$$R_{gold} = P_{1,t} - P_{2,t} \tag{1}$$

Bitcoin rose. This indicator shows how much the price of Bitcoin has changed on the current day compared to the previous day.

$$R_{bitcoin} = P_{2,t} - P_{2,t-1} \tag{2}$$

Gold 15-day average price. The price of gold fluctuates slightly, which is suitable for medium and long-term trading, so the paper averages the gold price on the current date and the previous 14 days to represent the "15-day moving average" indicator.

$$A_{gold} = \frac{\left(P_{1,t-14} + P_{1,t-13} + ... + P_{1,t-1} + P_{1,t}\right)}{15} \tag{3}$$

Bitcoin 5-day average price. The price of Bitcoin fluctuates greatly, which fits short-term trading, so the paper set the "five-day moving average" indicator as one of the important trading evaluation indicators. It calculates the average of the Bitcoin price on the current date and the previous four days to get the result.

$$A_{bitcoin} = \frac{\left(P_{2,t-4} + P_{2,t-3} + P_{2,t-2} + P_{2,t-1} + P_{2,t}\right)}{5} \tag{4}$$

A bull market is a period of optimistic growth in financial markets. It is often seen when favorable economic conditions reflect the confidence level in investors who would typically buy and hold during this period. A bear market is defined as a period of pessimism in which financial markets fall. People often lose confidence in the economy, and investors sell their investments, as they expect to pare losses from an expected decline. Simply put, a bull market can be defined as a generally bullish long-term stock market, and a bear market corresponds to a relatively long period of decline and higher volatility. There are also "bull and bear markets in the gold and bitcoin trading markets." From a macro perspective, traders make different decisions under different market conditions. Therefore, the

judgment of the bull and bear markets has an important influence on the final trading decisions. Most importantly, this article will establish the first mathematical model to judge whether the market is bull or bear.

3.2. Investment Risk Model

In the actual investment market transactions, this article will consider whether the overall market environment is a bull market or a bear market and consider other investment risks, and the measurement of these risks requires specific mathematical models. Therefore, this paper will establish an investment risk model to help predict subsequent investment strategies. The specific steps for calculating weights and overall scores are as follows:

First, normalize the data:

$$y_{ij} = \frac{x_{ij} - \min\limits_{1\le i\le n}\{x_{ij}\}}{\max\limits_{1\le i\le n}\{x_{ij}\} - \min\limits_{1\le i\le n}\{x_{ij}\}} \tag{5}$$

Then, calculate the entropy of the j-th index:

$$e_j = -\frac{1}{\ln 5}\sum_{1}^{n} p_{ij} \ln p_{ij} \tag{6}$$

Next, calculate the degree of dispersion of the index:

$$d_j = 1 - e_j \tag{7}$$

And then, the paper could calculate the weight of the jth indicator:

$$w_j = \frac{d_j}{\sum_{j=1}^{n} d_j} \tag{8}$$

Calculate the weighted value of each indicator;

$$z_{ij} = w_j y_{ij} \tag{9}$$

Afterward, calculate the distance of each indicator to the positive and negative ideal solutions:

$$d_i^+ = \sqrt{\sum_{j=1}^{n}\left(z_{ij} - p_j\right)^2}, i = 1,...n \tag{10}$$

$$d_i^- = \sqrt{\sum_{j=1}^{n}\left(z_{ij} - q_j\right)^2}, i = 1,...n \tag{11}$$

Finally, calculate the composite score of each indicator and normalize the scores:

$$F_i = \frac{d_i^-}{d_i^- + d_i^+} \tag{12}$$

3.3. ARIMA Series Prediction Model

Based on the indicators obtained from the above three models - bull and bear markets judgment model, investment risk model, and ARIMA series prediction model, combined with the innovative addition of "BOLL range," the paper finally determines this final total investment trading strategy model through dynamic programming.

3.4. Determination of BOLL Amplitude

The determination of the BOLL range comes from the BOLL indicator (Bollinger Bands), which uses statistical principles to find the standard deviation of the stock price and its confidence interval to determine the fluctuation range and future trend of the stock price and use the waveband to display the safe high and low price of the stock price. Furthermore, its upper and lower limits are not fixed and vary with the rolling stock price. Similar to the stock market, this paper measures the volatility of the gold and bitcoin trading markets through the BOLL indicator. The calculation formula is as follows:

$$MB_i = \frac{1}{15}\sum_{i}^{i+15} P_i, i = 1,...,1811 \tag{13}$$

$$UP_i = MB_i + k \times MD_i \tag{14}$$

$$DN_i = MB_i - k \times MD_i \tag{15}$$

$$MD_i = \sqrt{\frac{\sum_{i}^{i+15}\left(P_i - MB_i\right)^2}{14}} \tag{16}$$

MB_i is the i-th 15-day average price, UP_i is the i'th upper trajectory, DN_i is the i'th upper trajectory, and MD_i is the i'th 15-day price. The standard deviation of k generally takes 2.

The BOLL amplitude is the difference between the upper and lower rail lines.

$$BB_i = UP_i - DN_i, i = 1...,1811 \tag{17}$$

3.5. Calculate Buy Score

How should trading decisions be made with the knowledge of important metrics such as returns, residuals, and BOLL magnitudes? Here, this paper introduces a purchase score to quantify purchase likelihood. Its calculation formula is as follows:

$$Score = 10 \times G - r + \frac{1}{risk} + \frac{1}{BB} \tag{18}$$

The score is the buy score, G is the increase, r is the residual, risk is the risk coefficient, and BB is the BOLL range. It is calculated separately for gold and bitcoin, and the buy scores can be obtained as shown in Figure 1 and 2.

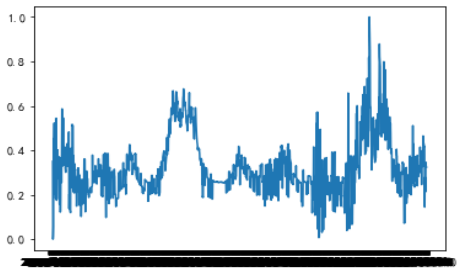

Figure 1. Gold's buy score

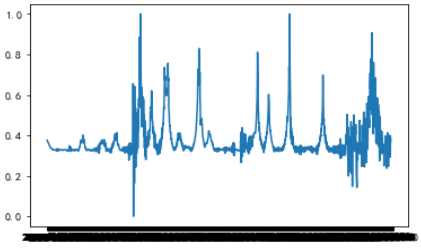

Figure 2. Bitcoin's buy score

Meanwhile, the larger buy score means a better market, which is more suitable for investment. Therefore, it can make decisions based on the buy score to determine the threshold value, define the purchase line of gold as Score1=0.58 and the throw line as Score2=0.3; define the purchase line of Bitcoin as Score3=0.70 and the throw line as Score2=0.50. The paper can get comparison charts of buying and selling gold and Bitcoin from this.

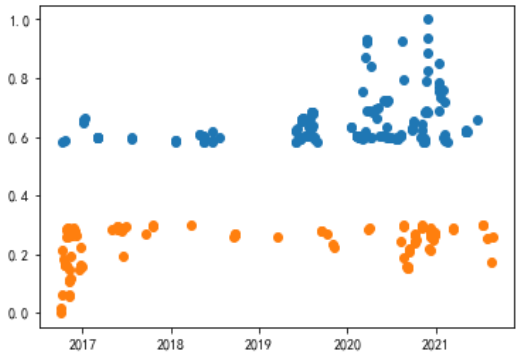

Figure 3. Chart of buying and selling gold

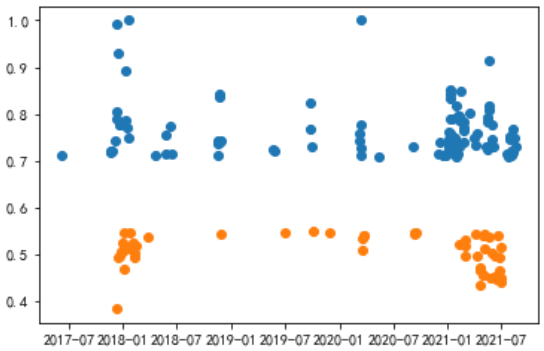

Figure 4. Chart of buying and selling bitcoin

4. ANALYSIS OF RESULTS

4.1. Calculate the Theoretical Maximum Profit

Based on the trading rules above, the paper uses python to perform dynamic programming to find the maximum profit. If the person only invests in Bitcoin, then by September 10, 2021, it will get a profit of $358,499.29; if the person only invests in gold, then by September 10, 2021, the person will have a profit of $6032.85. So, the volatility of Bitcoin is much greater than that of gold, but its income is very considerable. When only one product is allowed to be invested, the value of Bitcoin is about 59.42 times that of gold.

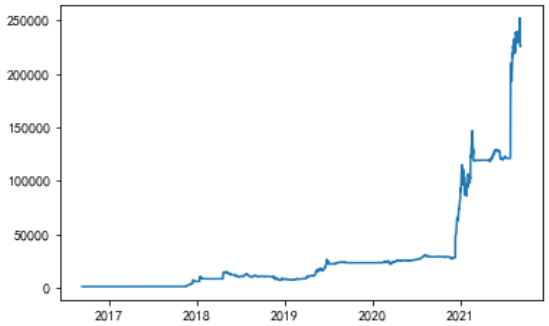

Figure 5. Trend of total assets

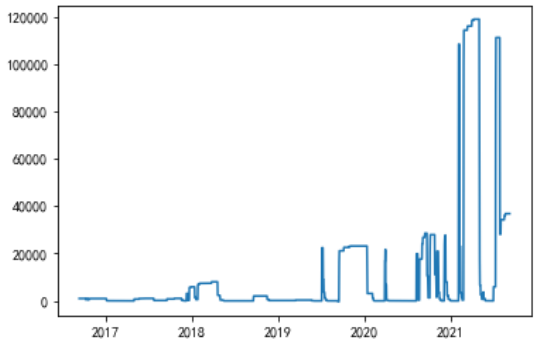

Figure 6. Trend of cash holding

Finally, the paper can get the total assets gold holding trend, which is shown in Figures 5 and 6. As can be seen from the figure, the total assets are on the rise, indicating that our model is scientific and in line with the expectations of maximizing profits and minimizing risks. According to the solution, the paper finally calculated that the initial investment value of $1,000 on September 10, 2021, is about $226,648.43, and some of the daily trading statuses are as follows.

Table.1. Transaction status

Date	Cash holdings	Total assets	Current gold profit	Current bitcoin profit
2021/9/6	36815.87	248717.85	8829.20	94409.25
2021/9/7	36815.88	252424.70	8828.74	101851.84
2021/9/8	36815.88	228446.16	8824.51	105562.92
2021/9/9	36815.88	225460.96	8821.00	81587.90
2021/9/10	36815.88	226648.42	8821.49	78602.21

4.2. Optimality of the model

In order to verify that the bull and bear market model we constructed is the best model, we use Perturbation Analysis (PA) to conduct a parameter optimization test of the model. After increasing 1% of the value of the parameter gold_cow, the result does not change significantly. So it is reasonable to believe the model is the best judgment model.

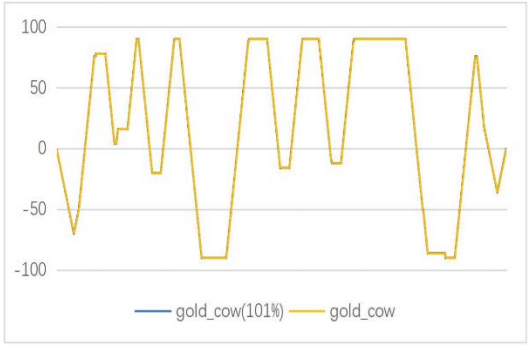

Figure 7. Comparison of numerical changes before and after PA

4.3. Sensitivity Analysis

To demonstrate the sensitivity of the model above, the paper presents the results of the testing of the model in the following sections. The paper uses the following formula in the calculation of the indicator "Buy Scores":

$$\text{Score} = \alpha G + \frac{\beta}{\text{risk}} + \frac{\beta}{BB} - r \tag{19}$$

We perform a sensitivity analysis on the adjustable parameters α and β in the formula. When we give different values to α and β, the total assets calculated by the model will also be different. We plot the results of total assets under the corresponding parameter values, shown in Figures 8 and 9.

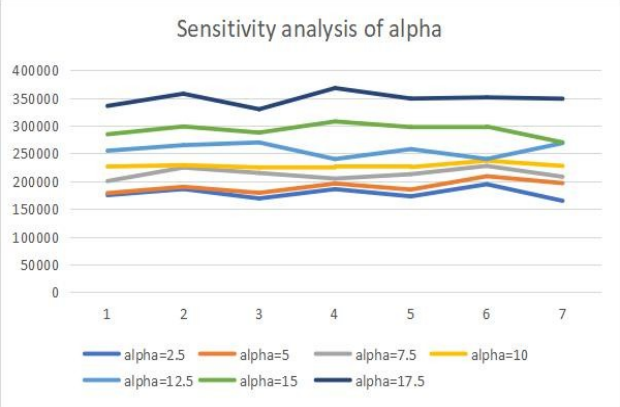

Figure 8. Sensitivity analysis of alpha

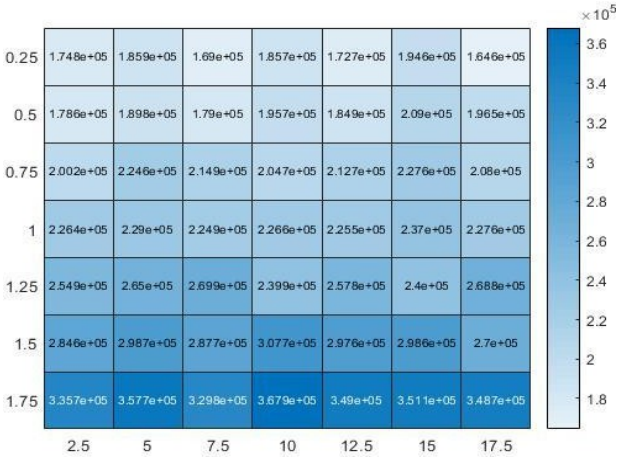

Figure 9. Heatmap of alpha and beta

According to the two images, the parameter α has strong robustness, while the parameter β has sensitivity.

5. Conclusion

In order to facilitate traders' decision-making in the gold and bitcoin markets, the paper proposes a novel trading model to maximize total returns, which is the MRAT Hybrid Model. The model is a hybrid transaction investment model based on the bull and bear market judgment model, investment risk model, and ARIMA time series forecast model. It can roughly judge whether the market is in a bull market or a bear market through the price increase and bias, which can better help traders judge the market situation. The optimal portfolio decision model in this paper obtains the purchased score according to the results calculated by the above model and then uses the method of dynamic programming to obtain the theoretical maximum profit. The final total portfolio investment assets are about $226648.43.

To verify that the MRAT hybrid model offers the best strategy, the paper use perturbation analysis to conduct a parameter optimization test of the bull and bear judgment model and trade strategy model. After increasing 1% of the value of the parameter gold_cow, the result does not change significantly. Therefore, it is reasonable to believe the model is the best judgment model.

References

[1] Wuyi Ye, Liping Sun & Baiqi Miao. (2020). Dynamic cointegration of gold and Bitcoin-Based on semi-parametric MIDAS loci regression model. Journal of Systems Science and Mathematical Sciences (07), 1270-12

[2] Hongjie Li (2018). Bitcoin-New “gold” in the Digital Age. China Business & Trade (02), 33-34.doi: 10.19699/j.cnki.issn2096-0298.2018.02.016.

[3] Kyriazis, N. A. (2020). Is Bitcoin Similar to Gold? An Integrated Overview of Empirical Findings. Journal of Risk and Financial Management, 13 (5), 88.

[4] Syed Jawad Hussain Shahzad, Elie Bouri, David Roubaud, Ladislav Kristoufek, Brian Lucey (2019). Is Bitcoin a better safe-haven investment than gold and commodities?. International Review of Financial Analysis, Volume

[5] Baur, Dirk & Dimpfl, Thomas & Kuck, Konstantin. (2017). Bitcoin, Gold and the US dollar-A Replication and Extension. Finance Research Letters. 25. 10.1016/j.frl.2017.10.012.

[6] Irene, H.& Perry, S. (2018). Can bitcoin replace gold in an investment portfolio?. Journal of Risk & Financial Management, 11 (3), 48.